1. Arithmetic Mean

The most popular and widely used measure of representing the entire data by one value is what most laymen call an 'average' and what the statisticians call the arithmetic mean. Its value is obtained by adding together all the items and by dividing this total by the number of items.

Simple Arithmetic Mean

a) Individual observations; (when the frequency is not given)

$$\overline{X} = \frac{X_1 + X_2 + X_3 + \dots\dots\dots + X_n}{N}$$

$$\overline{X} = \frac{\Sigma X}{N}$$

Where; $\overline{X}$ = Arithmetic mean

ΣX = Sum of all the values of the variable X

N = number of observations.

b) Discrete series;

$$\overline{X} = \frac{\Sigma f x}{N}$$

Where; f = frequency

X = variable

N = total number of observations.

c) Continuous series;

$$\overline{X} = \frac{\Sigma f m}{N}$$

Where; m = mid-point of various classes

f = frequency of each class

N = total frequency

Example 1: The following table gives the monthly income of 10 employees in an office;

Income (Rs.): 1780, 1760, 1690, 1750, 1840, 1920, 1100, 1810, 1050, 1950.

Calculate the arithmetic mean of incomes?

Solution:

$$\overline{X} = \frac{\Sigma X}{N}$$

$$= \frac{16650}{10}$$

$$\overline{X} = 1665$$

Hence, the average income is 1665 Rs.

Example 2: From the following data of marks obtained by 60 students of a class, calculate the arithmetic mean?

Marks	20	30	40	50	60	70
No. of students	8	12	20	10	6	4

Solution:

Marks (x)	No. of students (f)	f x
20	8	160
30	12	360
40	20	800
50	10	500
60	6	360
70	4	280
	N = 60	Σ f x = 2460

$$\overline{X} = \frac{\Sigma f x}{N}$$

$$= \frac{2460}{60}$$

$$\overline{X} = 41$$

Hence, the average marks = 41

Example 3: From the following data compute the arithmetic mean?

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	5	10	25	30	20	10

Solution:

Marks (x)	Midpoint (m)	No. of students (f)	f m
0-10	5	5	25
10-20	15	10	150
20-30	25	25	625
30-40	35	30	1050
40-50	45	20	900
50-60	55	10	550
		N = 100	Σ f m = 3300

Relationship among the averages

In any distribution when the original items differ in size the value of arithmetic mean (AM), geometric mean (GM) and harmonic mean (HM) would also differ and will be in the following order;

$$AM \geq GM \geq HM$$

i.e., arithmetic mean is greater than geometric mean and geometric mean is greater than the harmonic mean. The equality signs hold only if all the numbers x_1, x_2 .....x_n are identical.

MEDIAN

It refers to the middle value in a distribution. In case of median one half of the items in the distribution have a value the size of the median value or smaller and one half has a value the size of the median value or larger.

For individual series; arrange the data in ascending order of magnitude.

When number of observation (N) is odd;

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} \text{item}$$

When number of observation (N) is even;

$$\text{Median} = \frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2} \text{item}$$

For continuous series;

$$\text{Median} = L + \frac{\frac{N}{2} - c.f.}{f} \times i$$

Where,

L = Lower limit of the median class, *i.e.*, the class in which the middle item of the distribution lies

c.f. = Cumulative frequency of the class preceding the median class or sum of the frequencies of all classes lower than the median class

f = simple frequency of the median class

i = the class interval of the median class

Example 7: Compute the median from the following data;

Mid-value	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

Solution:

Since we are given the mid-values, we should find out the upper and lower limits of the various classes.

Class group	f	c.f.
100-120	6	6
120-130	25	31
130-140	48	79
140-150	72	151
150-160	116	267
160-170	60	327
170-180	38	365
180-190	22	387
190-200	3	390

$$\text{Median} = \text{Size of } \frac{N}{2}^{th} \text{ item} = \frac{390}{2} = 195^{th} \text{ item}$$

∴ Median lie in the class 150-160.

$$\text{Median} = L + \frac{\frac{N}{2} - c.f.}{f} \times i$$

$$= 150 + \frac{195 - 151}{116} \times 10$$

$$= 150 + 3.79$$

$$= 153.79$$

MODE

The mode or the modal value is that value in a series of observations which occurs with the greatest frequency. For example, the mode of the series 4, 3, 6, 8, 3, 10, 7, 3 would be 3, since this value occurs more frequently.

In individual observation, mode can be determined by counting the number of times the various values repeat themselves and the value occurring maximum number of times is the modal value.

In discrete series, quite often mode can be determined just by inspection, i.e., by looking to that value of the variable around which the items are most heavily concentrated.

Properties of mode

- The score that occurs most often and, therefore, the most typical value
- The only measure appropriate for unordered qualitative variables
- More appropriate than the mean or the median for quantitative variables that are inherently discrete.
- The easiest measure to compute
- Much more subject to sampling fluctuation that the mean and the median
- Less mathematically tractable than the mean and median
- Not necessarily existent, as when a distribution has two or more scores with the same maximum frequency.
- Rarely used in advanced statistical procedures.

Relationship between mean, median and mode

A distribution in which the values of mean, median and mode coincide (i.e., mean = median = mode) is known as symmetrical distribution. Conversely stated, when the values of mean, median and mode are not equal the distribution is known as asymmetrical or skewed.

Karl Pearson has expressed this relationship as follows;

$$\text{Mode} = \text{Mean} - (\text{Mean} - \text{Median})$$

$$\text{Mode} = 3\ \text{Median} - 2\ \text{Mean}$$

$$\text{Median} = \text{Mode} + 2/3\ (\text{Mean} - \text{Mode})$$